Anatomy and Phenomenology of the Lepton Flavor Universality in SUSY Theories

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Abstract

High precision electroweak tests, such as deviations from the Standard Model expectations of the Lepton Flavor Universality breaking in $K \to \ell\nu_\ell$ (with $l = e$ or μ), represent a powerful tool to test the Standard Model and, hence, to constrain or obtain indirect hints of New Physics beyond it. We explore such a possibility within Supersymmetric theories. Interestingly enough, a process that in itself does not need lepton flavor violation to occur, i.e. the violation of $\mu - e$ non-universality in $K_{\ell 2}$, proves to be quite effective in constraining not only relevant regions of SUSY models where lepton flavor is conserved, but even those where specific lepton flavor violating contributions arise. Indeed, a comparison with analogous bounds coming from τ lepton flavor violating decays shows the relevance of the measurement of R_K to probe Lepton Flavor Violation in SUSY. We outline the role and the interplay of the direct New Physics searches at the LHC with the indirect searches performed by LFU tests.

1 Introduction

The study of Lepton Flavor Universality (LFU) represents a powerful tool to test the Standard Model (SM) and, hence, to constrain or obtain indirect hints of new physics beyond it. Kaon and pion physics are obvious grounds where to perform such tests, for instance in the $\pi \to \ell\nu_\ell$ and $K \to \ell\nu_\ell$ decays, where $\ell = e$ or μ. In particular, defining $(R_P^{e/\mu})_{SM} = \Gamma(P \to e\nu_e)_{SM}/\Gamma(P \to \mu\nu_e)_{SM}$ and $(R_P^{e/\mu})_{exp.} = \Gamma(P \to e\nu)_{exp.}/\Gamma(P \to \mu\nu)_{exp.}$, the difference of the ratio

$$R_P^{e/\mu} = \frac{(R_P^{e/\mu})_{exp.}}{(R_P^{e/\mu})_{SM}} = 1 + \Delta r_P^{e/\mu} \tag{1}$$

from unit signals the presence of LFU violating New Physics (NP). Given that $(R_P^{e/\mu})_{SM}$ is accurately predicted, both for $P = \pi$ (0.02% accuracy [1]) and $P = K$ (0.04% accuracy [1]), it turns out that the determination of $(R_P^{e/\mu})$ constitutes a major precision test of the SM.

These precision tests are equally interesting and fully complementary to the flavor-conserving electroweak precision tests and to the FCNC tests performed in hadronic and leptonic physics (rare kaon, charm and B physics, lepton Flavor Violation (LFV)): the smallness of NP effects is more than compensated in terms of NP sensitivity by the excellent experimental resolution and the good theoretical control. The limiting factor in the determination $R_K^{e/\mu}$ is the $K \to e\nu$ rate, whose experimental knowledge has been quite poor so far.

The current world average $(R_K^{e/\mu})_{exp.} = (2.45 \pm 0.11) \times 10^{-5}$ [2] will be soon improved thanks to a series of preliminary results by NA48/2 and KLOE (see Fig. 1). The two results by NA48/2, being based on different data sets (2003 [3] and 2004 [3], respectively) with different running conditions, should be regarded as completely independent. Combining these new results with the PDG value yields [3]

$$(R_K^{e/\mu})_{exp.} = (2.457 \pm 0.032) \times 10^{-5} \ . \tag{2}$$

This result is in good agreement with the SM expectation and has a relative error ($\sim 1.3\%$) three times smaller compared to the previous world average. Further improvements in the knowledge of $(R_K^{e/\mu})_{exp.}$ would be more than welcome. Moreover, also the KLOE collaboration will reach an error down to the 1% level on $R_K^{e/\mu}$, once the remaining statistics will be added and the reconstruction efficiency improved [3].

Last but not least, an error on $(R_K^{e/\mu})_{exp.}$ of about 0.3% is the ambitious goal of the 2007 dedicated run of the CERN-P326 collaboration (the successor of NA48) [3]. If these expectations will be fulfilled, in a short term the error on the world average of $R_K^{e/\mu}$ will decrease by an additional factor of four.

In the following, we consider low-energy minimal SUSY extensions of the SM (MSSM) with R parity as the source of NP to be tested by $R_K^{e/\mu}$ [4]. As discussed in [4], it is indeed possible for regions of the MSSM to obtain $\Delta r_{NP}^{e-\mu}$ of $\mathcal{O}(10^{-2})$ and, such large contributions to $K_{\ell 2}$, do not arise from SUSY lepton flavor conserving (LFC) effects, but, rather, from lepton flavor violating (LFV) ones.

The main reason is that, whenever new physics acts in $K_{\ell 2}$ to create a departure from the strict SM $\mu - e$ universality, these new contributions will typically be proportional to the lepton masses. Hence, what occurs in the SUSY case is that LFC contributions are suppressed with respect to the LFV ones by higher powers of the first two generations lepton masses (it turns out that the first contributions to $\Delta r_{NP}^{e-\mu}$ from LFC terms arise at the cubic order in m_ℓ, with $\ell = e, \mu$). Instead, for the LFV contributions to $R_K^{e/\mu}$ one can select those which involve flavor changes from the first two lepton generations to the third one with the possibility of picking up terms proportional to the tau-Yukawa coupling

	$(R_K^{e/\mu})_{exp.}$ $[10^{-5}]$
PDG 2006 [2]	2.45 ± 0.11
NA48/2 '03 prel.	$2.416 \pm 0.043 \pm 0.024$
NA48/2 '04 prel.	$2.455 \pm 0.045 \pm 0.041$
KLOE prel.	$2.55 \pm 0.05 \pm 0.05$
SM prediction	2.472 ± 0.001

Figure 1: Current experimental data on $R_K^{e/\mu}$ from [3].

which can be large in the large $\tan\beta$ regime (the parameter $\tan\beta$ denotes the ratio of Higgs vacuum expectation values responsible for the up- and down- quark masses, respectively). Moreover, the relevant one-loop induced LFV Yukawa interactions are known [5] to acquire an additional $\tan\beta$ factor with respect to the tree level LFC Yukawa terms. Thus, the loop suppression factor can be (partially) compensated in the large $\tan\beta$ regime.

In this paper, we analise the domain of $\Delta r_K^{e/\mu}$ between 10^{-2} and 10^{-3}. We show that:

i) if $\Delta r_K^{e/\mu}$ is found to be $\Delta r_K^{e/\mu} \geq 5 \times 10^{-3}$, then the signal unambiguously indicates the presence of LFV sources.

ii) if $\Delta r_K^{e/\mu} \leq 5 \times 10^{-3}$, then both the LFC and LFV sources can account for the effect.

iii) a value of $\Delta r_K^{e/\mu}$ between 5×10^{-3} and 10^{-3} severely constrains the parameter space in the $M_H - \tan\beta$ plane.

iv) if a signal exists at a such a level, the LHC results become the crucial tool to discriminate between the LFC and LFV sources of LFU breaking.

v) there exists a strong correlation between large LFU violation and LFV in lepton decays (mainly τ decays); another interesting relation concerns the regions of SUSY parameter space where the deviation from the SM expectation for the muon anomalous magnetic moment finds a SUSY explanation and that allowing for a sizeable LFU violation.

The paper is organized as follow: in Section 2, we outline general considerations about LFU in P_{l2}. In section Section 3, we specialize to the LFV case while in Section 4, we discuss the additional possibility of LFC contributions. In Section 5, we list the constraints we have imposed on the SUSY parameter space before starting the analysis of the LFU breaking effects. In Section 6, we discuss the correlation between LFU violation and LFV in lepton decays and their possible connection with a SUSY explanation for the anomalous magnetic moment of the muon. In Section 7, we present the quantitative analysis of our results incorporating the constraints of the above sections. In Section 8, we extend the analysis of LFU breaking effects to a generic two Higgs Doublet Model with tree level flavor changing interactions between the Higgs bosons and the fermions.

Finally, in Section 9 we summarize the main results of the present analysis.

2 Lepton Flavor Universality in $P_{\ell 2}$

Within the SM, possible departures from the LFU are predicted to be

$$|\Delta r_{\rm SM}^{\ell_1/\ell_2}| = \mathcal{O}[(\alpha/4\pi)\times(m^2_{\ell_{1(2)}}/M^2_W)]\,, \tag{3}$$

and thus completely negligible. This explains why the study of LFU breaking represents a very useful tool to look for NP effects.

On general grounds, violations of LFU in charged current interactions can be classified into two classes: i) corrections to the strength of the effective $(V-A)\times(V-A)$ four-fermion interaction, ii) four-fermion interactions with new Lorentz structures.

As an example of the first class, we mention the $W\ell\nu_\ell$ vertex correction through a loop of new particles: the induced effect is of order $(\alpha/4\pi)\times(M^2_W/M^2_{NP})$, hence unobservably small. Second class is definitely more promising: the typical example is the scalar current induced by tree level Higgs exchange, with mass-dependent coupling ($H\ell\nu\sim m_\ell\tan\beta$).

In the following, we will analyze LFU breaking effects arising from this latter class occurring in $P_{\ell 2}$.

Due to the V-A structure of the weak interactions, the SM contributions to $P_{\ell 2}$ are helicity suppressed; hence, these processes are very sensitive to non-SM effects (such as multi-Higgs effects) which might induce an effective pseudoscalar hadronic weak current.

In particular, charged Higgs bosons ($H^\pm$) appearing in any model with two Higgs doublets (including the SUSY case) can contribute at tree level to the above processes.

The relevant four-Fermi interaction for the decay of charged mesons induced by $W^\pm$ and $H^\pm$ has the following form:

$$\frac{4G_F}{\sqrt{2}}V_{ud}\left[(\overline{u}\gamma_\mu P_L d)(\,\overline{l}\gamma^\mu P_L\nu_l\,)+\Delta^{ij}\,t^2_\beta\left(\frac{m_d m_{l_i}}{m^2_{H^\pm}}\right)(\,\overline{u}P_R d)(\,\overline{l}_i P_L\nu_j\,)\right]\,, \tag{4}$$

where $P_{R,L}=(1\pm\gamma_5)/2$ and we kept only the t_β (with $t_\beta=\tan\beta$) enhanced part of the $H^\pm ud$ coupling, namely the $m_d t_\beta$ term.

The quantity $\Delta^{ij}=\Delta^{ij}(\delta^{ij},\tan\beta,m_{l_j},\tilde{m})$ may depend, in general, on the mixing angle δ^{ij} regulating the flavor transition ij, on the $\tan\beta$ parameter, on the masses of all charged lepton generations and, finally, on all the possible susy masses $\tilde{m}$ determining the effective vertex.

The decays $P\to\ell\nu$ ($P=K,\pi$) proceed via the axial-vector part of the $W^\pm$ coupling and via the pseudoscalar part of the $H^\pm$ coupling. Then, once we implement the PCAC's

$$<0|\overline{u}\gamma_\mu\gamma_5 d|M^-> = if_M p^\mu_M \quad , \quad <0|\overline{u}\gamma_5 d|M^-> = -if_M\frac{m^2_M}{m_d+m_u}\,, \tag{5}$$

it is found that

$$R_{P\ell_i\nu} = \left[1 - \Delta^{ii}\left(\frac{m_{d_P}}{m_{d_P}+m_{u_P}}\right)\frac{m_P^2}{M_{H^+}^2}t_\beta^2\right]^2 + \Sigma_{j\neq i}|\Delta^{ij}|^2\left(\frac{m_{d_P}}{m_{d_P}+m_{u_P}}\right)^2\frac{m_P^4}{M_{H^+}^4}t_\beta^4 \qquad (6)$$

The tree level charged Higgs exchange leads to a contribution with $i = j$ and $\Delta^{ii} = 1$. However, the introduction of a charged scalar current (induced by a H^+) does not introduce any deviation from the SM expectation of the LFU breaking in $R_P^{e/\mu}$.

Indeed, we observe that the SM amplitude is proportional to m_ℓ because of the helicity suppression while the charged Higgs one carries the m_ℓ dependence through the Yukawa coupling.

As a result, the first SUSY contributions violating the $\mu - e$ universality in $P \to \ell\nu$ decays arise at the one-loop level with various diagrams involving exchanges of (charged and neutral) Higgs scalars, charginos, neutralinos and sleptons. For our purpose, it is relevant to divide all such contributions into two classes:

i) LFC contributions, where the charged meson M decays without FCNC in the leptonic sector, i.e. $P \to \ell\nu_\ell$;

ii) LFV contributions $P \to \ell_i\nu_k$, with i and k referring to different generations (in particular, the interesting case will be for $i = e, \mu$, and $k = \tau$).

In the following sections we address separately the case of LFC and LFV contributions.

3 The lepton flavor violating case

Within SUSY theories, there exist two different classes of LFV interactions:

i) Gauge-mediated LFV interactions,

ii) Higgs-mediated LFV interactions.

As regards the class *i*), LFV effects are induced by the exchange of gauginos and sleptons; these contributions decouple with the heaviest mass m_{SUSY} circulating in the slepton/gaugino loops.

Concerning the case *ii*), we remind that models containing at least two Higgs doublets generally allow flavor violating couplings of the Higgs bosons with the fermions [6]. However in the MSSM such LFV couplings are absent at tree level since we have one higgs doublet coupling uniquely to the up-sector, while the other higgs doublet couples only to the down-sector. However, once non holomorphic terms are generated by loop effects (so called HRS corrections [7]) and given a source of LFV among the sleptons, Higgs-mediated $H\bar{\ell}_i\ell_j$ LFV couplings are unavoidable [5]. These effects decouple with the heavy Higgs mass scale m_H but they do not decouple with the mass scale of the sleptons/gauginos circulating in the loop, given that the effective LFV Yukawa couplings

arise from dimension four operators. As it is well known, higgs mediated effects to rare decays start being competitive with the gaugino mediated ones when m_{SUSY} is roughly one order of magnitude heavier then m_H and for $\tan\beta \sim \mathcal{O}(50)$ [8]. On general ground, there is no reason to assume that $m_H \simeq m_{SUSY}$, unless specific models of SUSY breaking are assumed.

We stress that the quantity which is determined experimentally and accounts for the deviation from the $\mu - e$ universality is

$$(R_P^{e/\mu})_{exp.} = \frac{\sum_i \Gamma(P \to e\nu_i)}{\sum_i \Gamma(P \to \mu\nu_i)} \qquad\qquad i = e, \mu, \tau. \tag{7}$$

with the sum extended over all (anti)neutrino flavors. In fact, experimentally, it is possible to measure only the charged lepton flavor in the decay products.

The dominant SUSY contributions to $R_P^{e/\mu} = (R_P^{e/\mu})_{exp.}/(R_P^{e/\mu})_{SM}$ arise from the charged Higgs exchange.

One could naively think that the SUSY effects in the LFV channels $P \to \ell_i \nu_k$ are further suppressed with respect to the LFC ones. On the contrary, charged Higgs mediated LFV contributions, in particular in the kaon decays into an electron or a muon and a tau neutrino, can be strongly enhanced.

In particular, the expressions for the effective couplings Δ^{ij} in Eq. 4 read

$$\Delta^{\ell\ell} = \frac{1}{(1+\epsilon t_\beta)(1+\epsilon_\ell t_\beta)} + \frac{m_\tau}{m_\ell}\frac{\Delta_{RL}^{\ell\ell} t_\beta}{(1+\epsilon t_\beta)(1+\epsilon_\tau t_\beta)^2} \tag{8}$$

$$\Delta^{\ell\tau} = \frac{m_\tau}{m_\ell}\frac{\Delta_R^{3l} t_\beta}{(1+\epsilon t_\beta)(1+\epsilon_\tau t_\beta)^2} \qquad\qquad l = e, \mu\,. \tag{9}$$

The first term in Eq. 8 refers to a tree level charged Higgs exchange while the second one stems from a double source of LFV that, as a final result, preserves the flavor. On the contrary, the contributions of Eq. 9 refer to LFV channels. Notice that the (loop induced) contributions arising from LFV sources is enhanced by the factor m_τ/m_ℓ (compared to the contributions from a tree level charged Higgs exchange) when the electron or muon in $(R_P^{e/\mu})_{exp.}$ are accompanied by a tau neutrino.

In the above expressions, we have also included the threshold corrections (proportional to ϵ, ϵ_ℓ, with $\epsilon \sim \alpha_s/4\pi$ and $\epsilon_\ell \sim \alpha_2/4\pi$)) for the quark and lepton yukawas appearing when we integrate out heavy degrees of freedom from the low energy effective theory [7].

In particular, a relevant observation for the following analysis is that the one-loop induced $\epsilon_\ell = \epsilon_\ell(m^2_{\tilde{\ell}}, M^2_{\tilde{\chi}})$ resummation factors carry a lepton flavor dependence through the slepton masses. Thus, as we will see in the next section, if the slepton generations have different masses, the ϵ_ℓ factors will generate a breaking of the LFU in low-energy observables.

The $\Delta^{3\ell}_{R,RL}$ terms are induced at one loop level by the exchange of Bino or Bino-Higgsino and sleptons. Since the Yukawa operator is of dimension four, the quantities

$\Delta_R^{3\ell}$ depend only on ratios of SUSY masses, hence avoiding SUSY decoupling. In the so called mass insertion (MI) approximation, the expressions of $\Delta_{R,RL}^{3\ell}$ are given by:

$$\Delta_R^{3\ell} \simeq \frac{\alpha_Y}{8\pi}\mu M_1 m_R^2 \delta_{RR}^{3\ell}\left[I'(M_1^2,\mu^2,m_R^2)-(\mu\leftrightarrow m_L)\right] \tag{10}$$

$$\Delta_{RL}^{\ell\ell} \simeq -\frac{\alpha_Y}{16\pi}\mu M_1 m_L^2 m_R^2\, \delta_{RR}^{\ell 3}\delta_{LL}^{3\ell}\, I''(M_1^2,m_L^2,m_R^2)\,, \tag{11}$$

where μ is the the Higgs mixing parameter, M_1 is the Bino ($\tilde{B}$) mass and $m_{L(R)}^2$ stands for the left-left (right-right) slepton mass matrix entry. The LFV MIs, i.e. $\delta_{XX}^{3\ell}=(\tilde{m}_\ell^2)_{XX}^{3\ell}/m_X^2$ $(X=L,R)$, are the off-diagonal flavor changing entries of the slepton mass matrix. The loop function $I'(x,y,z)$ is such that $I'(x,y,z) = dI(x,y,z)/dz$, where $I(x,y,z)$ refers to the standard three point one-loop integral which has mass dimension -2; morever, $I''(x,y,z) = d^2I(x,y,z)/dydz$. As it is clearly shown by Eq. 10, $\Delta_R^{3\ell}$ vanishes for $\mu = m_L$. On the other hand, both $\Delta_R^{3\ell}$ and $\Delta_{RL}^{3\ell}$ assume the their maximum values when $\mu \gg M_1, m_L, m_R$; this is easily understood reminding that Higgs mediated effects originate from non holomorphic corrections that are driven by the $\mu H_1 H_2$ term in the superpotential.

In particular, in the limit where $\mu \gg \tilde{m} = M_1 = m_L = m_R$, it turns out that $\Delta_R^{3\ell} \simeq \alpha_Y/16\pi \times \mu/\tilde{m} \times \delta_{RR}^{3\ell}$ and $\Delta_{RL}^{\ell\ell} \simeq \alpha_Y/32\pi \times \mu/\tilde{m} \times \delta_{RR}^{\ell 3}\delta_{LL}^{3\ell}$ [1]. Making use of the effective couplings of Eq. (9), it turns out that the dominant contribution to $\Delta r_{NP}^{e-\mu}$ reads

$$R_K^{e/\mu} \simeq \left|1-\frac{m_K^2}{M_H^2}\frac{m_\tau}{m_e}\frac{\Delta_{RL}^{11}t_\beta^3}{(1+\epsilon t_\beta)(1+\epsilon_\tau t_\beta)^2}\right|^2 + \left(\frac{m_K^4}{M_H^4}\right)\left(\frac{m_\tau^2}{m_e^2}\right)\frac{|\Delta_R^{31}|^2 t_\beta^6}{(1+\epsilon t_\beta)^2(1+\epsilon_\tau t_\beta)^4}. \tag{12}$$

In the above expression, we have included the interference between SM and SUSY LFC terms (arising from a double LFV source). In Eq. (12) terms proportional to Δ_R^{32} are neglected given that they are suppressed by a factor m_e^2/m_μ^2 with respect to the term proportional to Δ_R^{31}.

Taking $\Delta_R^{31} \simeq 5\cdot 10^{-4}$ (by means of a numerical analysis, it turns out that $\Delta_R^{3\ell} \leq 10^{-3}$ [9]), $\tan\beta = 40$ and $M_H = 500GeV$ we end up with $\Delta r_{K\,SUSY}^{e-\mu} \simeq 10^{-2}$. We see that in the large (but not extreme) $\tan\beta$ regime and with a relatively heavy $H^\pm$, it is possible to reach contributions to $\Delta r_{K\,SUSY}^{e-\mu}$ at the percent level thanks to the possible LFV enhancements arising in SUSY models.

Turning to pion physics, one could wonder whether the analogous quantity $\Delta r_{\pi\,SUSY}^{e-\mu}$ is able to constrain SUSY LFV. However, the correlation between $\Delta r_{\pi\,SUSY}^{e-\mu}$ and $\Delta r_{K\,SUSY}^{e-\mu}$:

$$\Delta r_{\pi\,SUSY}^{e-\mu} \simeq \left(\frac{m_d}{m_u+m_d}\right)^2\left(\frac{m_\pi^4}{m_k^4}\right)\Delta r_{K\,SUSY}^{e-\mu} \tag{13}$$

clearly shows that the constraints on $\Delta r_{K\,susy}^{e-\mu}$ force $\Delta r_{\pi\,susy}^{e-\mu}$ to be much below its current experimental upper bound.

[1] $\mathrm{Im}(\delta_{RR}^{13}\delta_{LL}^{31})$ is strongly constrained by the electron electric dipole moment [10]. However, sizable contributions to R_K^{LFV} can still be induced by $\mathrm{Re}(\delta_{RR}^{13}\delta_{LL}^{31})$.

4 The lepton flavor conserving case

We now reconsider Eq. 4 in the $i = j$ case, i.e. the lepton flavor conserving channels. In absence of LFV interactions, Δ_{ii} reads $\Delta_{ii} = 1/[(1+\epsilon t_\beta)(1+\epsilon_\ell t_\beta)]$. This leads to

$$\frac{\Gamma(P\to\ell\nu)}{\Gamma(P\to\ell\nu)_{SM}} = \left[1 - \left(\frac{m_{d_P}}{m_{d_P}+m_{u_P}}\right)\frac{m_P^2}{M_{H^+}^2}\frac{t_\beta^2}{(1+\epsilon t_\beta)(1+\epsilon_\ell t_\beta)}\right]^2 \tag{14}$$

As discussed in the previous section, tree level H^+ contributions do not introduce any breaking of the LU in $R_P^{e/\mu}$. However, this is strictly true only if $\epsilon_e = \epsilon_\mu$, as clearly shown by Eq. 14. In particular, for non universal slepton masses, it turns out that $\epsilon_e \neq \epsilon_\mu$ (remind that $\epsilon_\ell = \epsilon_\ell(m_{\tilde{\ell}}^2, M_{\tilde{\chi}}^2)$), and LFU breaking effects are generated. By means of Eq.14, we find that $\Delta r_P^{e/\mu}$ is well approximated by the following expression

$$\Delta r_P^{e/\mu} \simeq -2\left(\frac{m_{d_P}}{m_{d_P}+m_{u_P}}\right)\frac{m_P^2}{M_{H^+}^2}\frac{t_\beta^3}{(1+\epsilon t_\beta)}(\epsilon_e-\epsilon_\mu)\,, \tag{15}$$

where we have assumed that $m_P^2/M_{H^+}^2 \ll 1$ and $\epsilon_\ell t_\beta \ll 1$. We observe that the NP sensitivity to the above effects of $K\to\ell\nu$ is higher than that of $\pi\to\ell\nu$ by a factor of $\Delta r_K^{e/\mu}/\Delta r_\pi^{e/\mu} \sim m_K^2/m_\pi^2$. The current experimental resolutions on these modes imply that $K\to\ell\nu$ is the best probe of the above scenario.

Moreover, LFC contributions to R_π and R_K can be also induced at the loop level by box, wave function renormalization and vertex contributions from SUSY particle exchange [4]. The complete calculation of the μ decay in the MSSM [11, 12] can be easily applied to the meson decays. The dominant contributions to $\Delta r_{SUSY}^{e-\mu}$ arise from the charginos/neutralinos sleptons ($\tilde{l}_{e,\mu}$) exchange and it has the form [4]

$$\Delta r_{SUSY}^{e-\mu} \sim \frac{\alpha_2}{4\pi}\left(\frac{\tilde{m}_\mu^2-\tilde{m}_e^2}{\tilde{m}_\mu^2+\tilde{m}_e^2}\right)\frac{m_W^2}{M_{SUSY}^2}, \tag{16}$$

thus, $\Delta r_{SUSY}^{e-\mu}$ can be of order $\Delta r_{SUSY}^{e-\mu} \leq 10^{-3}$, provided there exists a large mass splitting among sleptons ($\tilde{m}_\mu^2 \ll \tilde{m}_e^2$ or $\tilde{m}_\mu^2 \gg \tilde{m}_e^2$) and a SUSY mass scale M_{SUSY} not much above the EW scale, i.e. $M_{SUSY} \sim m_W$. So, it turns out that all these LFC contributions yield values of $\Delta r_{K\,SUSY}^{e-\mu}$ which are smaller than the current and expected future experimental sensitivities in kaon physics.

On the other hand, given that the NP sensitivity to the above effects of $\Delta r_K^{e/\mu}$ and $\Delta r_\pi^{e/\mu}$ is the same and since the experimental resolution is better in the pion system, for this flavor conserving SUSY contribution it is the decay $\pi\to\ell\nu$ to represent the best place where to look for LFU violation. In particular,

$$R_\pi^{exp.} = (1.230 \pm 0.004)\cdot 10^{-4} \quad \text{PDG} \tag{17}$$

and by making a comparison with the SM prediction

$$R_\pi^{SM} = (1.2354 \pm 0.0002)\cdot 10^{-4} \tag{18}$$

one obtains (at the 2σ level)

$$-0.0107 \leq \Delta r_{NP}^{e-\mu} \leq 0.0022. \tag{19}$$

Comparing this interval for $\Delta r_{NP}^{e-\mu}$ with the above value of $\Delta r_{SUSY}^{e-\mu}$, it turns out that only under rather particular conditions (very large mass splitting of sleptons of different generation, relatively light SUSY scale) can one obtain visible LFC SUSY contributions to the LFU violation in pion decays [13].

5 Constraints

In this section, we list the constraints we have imposed on the SUSY parameter space before starting the analysis of the LU breaking effects.

5.1 Direct SUSY search

The framework in which we work is a low-energy R-parity conserving susy model with generic LFV soft breaking terms. We perform a scan up to a mass scale of 5TeV of the following low energy parameters: the gaugino masses M_i $(i = 1, 3)$, the μ term, the left-left and right-right sfermion mass terms for the first two and the third generations $M_{\tilde{f}}$, the trilinear coupling in the stop sector A_t; moreover $\tan\beta < 60$. At the low scale, we impose the following constraints on each point:

- Lower bound on the light and pseudo–scalar Higgs masses [14];
- The LEP constraints on the lightest chargino and sfermion masses [15];
- The LEP and Tevatron constrains on squarks, gluino and charged Higgs masses [15]
- Absence of charge and/or colour breaking minima [16].
- The lightest susy particle (LSP) is neutral.
- Electroweak Precision Observables (EWPO) constraints [17].

For future convenience, it is useful to recall which are the necessary conditions under which the lightest Higgs mass bounds are satisfied. First of all, we remind that the LEPII bound $(m_h)_{\rm SM} \gtrsim 114$ GeV, applies also to SUSY theories, irrespective of the $\tan\beta$ values, provided we assume the decoupling regime, roughly implying that $M_{H,A} \geq 200GeV$. Indeed, this will represent the case under study in the present work. Within the MSSM, the lightest Higgs mass is bounded from above. In particular, we can write $m_h^2 = m_h^{2({\rm tree})} +$

$m_h^{2(\text{loop})}$ where, for large $\tan\beta$, $m_h^{2(\text{tree})} \sim m_Z^2 - 4m_Z^2 m_A^2/(m_A^2 - m_Z^2)\cot^2\beta$. The most significant loop contribution is given by

$$m_h^{2(\text{loop})} = \frac{3m_t^4}{4\pi^2 v^2}\left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) + \frac{1}{2}\left(\frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\right)^2\left(2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\right)\right], \tag{20}$$

where $X_t = A_t - \mu^*\cot\beta$. Thus, the tree level contribution, that is maximum for moderate to large $\tan\beta$, has to be supplemented by sizable loop corrections. In particular, if the stop mixing is small, $|X_t/m_{\tilde{t}_{1,2}}|^2 \ll 1$, the correction depends only on the logarithm of the stop masses, so these must be rather heavy. If, however, the stop mixing is large, much lighter stops can still yield large loop corrections. However, as we will see, this last possibility is disfavored by the $b \to s\gamma$ constraints, specially in the large $\tan\beta$ regime.

5.2 B-physics observables

5.2.1 $\mathcal{B}(B \to X_s\gamma)$

As it is well known, $\mathcal{B}(B \to X_s\gamma)$ is a particularly sensitive observable to possible non-standard contributions and it provides a non-trivial constraint on the SUSY mass spectrum given its precise experimental determination and the very accurate SM calculation at the NNLO [18]. According to the recent NNLO analysis of Ref. [18], the SM prediction is $\mathcal{B}(B \to X_s\gamma; E_\gamma > 1.6\ \text{GeV})^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$. Combining this result with the experimental average [19–21] $\mathcal{B}(B \to X_s\gamma; E_\gamma > 1.6\ \text{GeV}))^{\text{exp}} = (3.55 \pm 0.24) \times 10^{-4}$ we obtain

$$R_{Bs\gamma} = \frac{\mathcal{B}^{\text{exp}}(B \to X_s\gamma)}{\mathcal{B}^{\text{SM}}(B \to X_s\gamma)} = 1.13 \pm 0.12\,. \tag{21}$$

In our numerical analysis we impose the above constraint at the 2σ C.L.

Within Minimal Flavor Violating (MFV) frameworks [22], the dominant SUSY contributions to $\mathcal{B}(B \to X_s\gamma)$ arise from the one-loop charged-Higgs and chargino-squark amplitudes. Charged-Higgs effects unambiguously increase the rate compared to the SM expectation, while the chargino-squark ones can have both signs depending on the sign of $sign(\mu A_{\tilde{t}})$. In this work we choose $\mu > 0$ (that is also preferred by the $(g-2)_\mu$ constraints) and $sign(A_{\tilde{t}}) < 0$, which implies destructive interference between chargino and charged Higgs contributions. A simple expression accounting for NP contributions in $B \to X_s\gamma$ is provided by [18, 23]

$$R_{Bs\gamma} \simeq 1 - 2.54\, C_7^{NP}(M_W) - 0.60\, C_8^{NP}(M_W) \tag{22}$$

where $C_{7,8}^{NP} = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^\pm}$ are the relevant Wilson coefficients for the New Physics contributions evaluated at the M_W scale. In particular, starting from the full expressions for

$C^{NP}_{7,8}$ of Ref. [24], one can derive the following approximate expressions

$$
\begin{aligned}
C_7^{H^\pm} &\simeq \left(\frac{1-\epsilon\, t_\beta}{1+\epsilon\, t_\beta}\right) \frac{m_t^2}{M^2_{H^\pm}}\, F^7_{H^\pm}\left(\frac{m_t^2}{M^2_{H^\pm}}\right) , \\
C_7^{\tilde{\chi}^\pm} &\simeq -\frac{A_{\tilde t}}{\mu}\frac{m_t^2}{\mu^2}\frac{t_\beta}{1+\epsilon\, t_\beta}\, F^7_{\tilde{\chi}^\pm}\left(\frac{m^2_{\tilde q}}{\mu^2}\right) ,
\end{aligned}
\tag{23}
$$

where $\epsilon \sim 10^{-2}$ for a degenerate SUSY spectrum and $F^7_{\tilde{\chi}^\pm}(1) \simeq 0.07$, $F^7_{\tilde{\chi}^\pm}(x \gg 1) \simeq (13/12 - 1/2\log(x))/x^2$ and $F^7_{\tilde{\chi}^\pm}(x \ll 1) \simeq 7/12 + 2/3\log(x)$ while $F^7_{H^\pm}(x_{tH}) \simeq 1/4 + 1/3\log(x_{tH})$ for $x_{tH} = m_t^2/M^2_{H^\pm} \ll 1$ and $F^7_{H^\pm}(1) \simeq -0.2$. We observe that, when $m_{\tilde q}/\mu \ll 1$, the lower bound on $m_{\tilde q}$ is set by the experimental limits on the lightest stop mass $m^2_{\tilde t_1} \simeq m^2_{\tilde q} - m_t|A_{\tilde t}|$ and on the sbottom mass $m^2_{\tilde b_1} \simeq m^2_{\tilde q} - m_b|\mu|t_\beta$. Similar expressions for the subleading contributions proportional to C_8^{NP} are not shown, although included in our numerical analysis.

Taking $C_7^{H^\pm}$ and $C_7^{\tilde\chi^\pm}$ separately, the following observations follow: i) the lower bound on $m_{H^\pm} \geq 295$GeV, holding at the 2σ level within a 2HDM framework (where it is assumed $\epsilon = 0$ and where $C_7^{\tilde\chi^\pm} = 0$), can be significantly relaxed within SUSY scenarios thanks to a reduction of $C_7^{H^\pm}$ driven by the threshold corrections ϵ; in particular if $\tan\beta \sim 50$ it turns out that $m_{H^\pm} \geq 200$GeV ii) for a natural scenario where all the SUSY masses have comparable size, in particular for $A_{\tilde t}/(\mu, m_{\tilde q}) \sim 1$, the regime $\tan\beta \sim 50$ necessarily implies that μ and/or $m_{\tilde q}$ lie in the ≥ 1TeV scale iii) the simultaneous requirement of large values for $\tan\beta$ and relatively light $m_{\tilde q}, \mu$ (below the 1TeV scale) necessarily implies either large cancellations between $C_7^{H^\pm}$ and $C_7^{\tilde\chi^\pm}$ and/or $A_{\tilde t}/(\mu, m_{\tilde q})$ significantly less than 1. However, as we have seen before, the scenario with relatively light $m_{\tilde q}$ and $A_{\tilde t}/m_{\tilde q} \ll 1$ is constrained by the lower bound on the lightest Higgs mass m_h.

5.2.2 $B \to \tau\nu$

The recent Belle [25] and BaBar [26] results for $B \to \ell\nu$ leads to the average $\mathcal{B}(B \to \tau\nu)^{\rm exp} = (1.42 \pm 0.43) \times 10^{-4}$. This should be compared with the SM expectation $\mathcal{B}(B \to \tau\nu)^{\rm SM} = G_F^2 m_B m_\tau^2 f_B^2 |V_{ub}|^2 (1 - m_\tau^2/m_B^2)^2 \tau_B/8\pi$, whose numerical value suffers from sizable parametrical uncertainties induced by f_B and V_{ub}. Taking $\tau_B = (1.643 \pm 0.010)$ps, $V_{ub} = (4.00 \pm 0.26) \times 10^{-3}$ and $f_B = 0.216 \pm 0.022$GeV [27], the best estimate is $\mathcal{B}(B \to \tau\nu)^{\rm SM} = (1.33 \pm 0.23) \times 10^{-4}$, which implies

$$
R^{\rm exp}_{B\tau\nu} = \frac{\mathcal{B}^{\rm exp}(B \to \tau\nu)}{\mathcal{B}^{\rm SM}(B \to \tau\nu)} \;=\; 1.07 \pm 0.37 \; . \tag{24}
$$

From the theoretical side, the $B \to \ell\nu$ process is one of the cleanest probes of the large $\tan\beta$ scenario due to its enhanced sensitivity to tree-level charged-Higgs exchange [28,29]. In particular, a scalar charged current induced by NP theories with extended Higgs sectors,

provides the following effects:

$$R_{B\ell\nu} = \left[1 - \frac{m_B^2}{M_{H^+}^2}\frac{t_\beta^2}{(1+\epsilon t_\beta)(1+\epsilon_\ell t_\beta)}\right]^2 \tag{25}$$

where we have included corrections both for the quark and lepton yukawas arising within SUSY theories.

The new physics effect on $R_{K\mu\nu} = \Gamma^{\rm SUSY}(K\to\mu\nu)/\Gamma^{\rm SM}(K\to\mu\nu)$ is obtained from Eq. 25 with the replacement $m_B^2 \to m_K^2$ [29]. Although the charged Higgs contributions are now suppressed by a factor $m_K^2/m_B^2 \simeq 1/100$, this is well compensated by the excellent experimental resolution [3] and the good theoretical control. However, given that these new physics effects are, in the most favorable cases, at the % level, we would need a theoretical prediction for the SM contribution at the same level to use this decay as an effective constraint. We would then need an independent determination both of f_K (possibly from lattice QCD) and V_{us} with such a level of accuracy.

The best strategy to fully exploit the New Physics sensitivity of K_{l2} systems is to consider the ratio $R^{'} = R_{K\mu\nu}/R_{\pi\mu\nu}$ [3, 29]. In fact, on the one side $R^{'}$ and $R_{K\mu\nu}$ have the same New Physics content (being $R_{\pi\mu\nu}$ not sizably affected by charged current interactions) and, on the other side, $R^{'}$ depends on $(f_K/f_\pi)^2$ instead of f_K^2 with f_K/f_π being much better under control compared to f_K by means of lattice QCD. However, at present, unquenched lattice calculations of f_K/f_π are still not well established. Therefore, although it may play a relevant role in the future, we do not include the constraints from $K\to l\nu$ in the present analysis.

The above argument for $K\to l\nu$ does not apply to $B\to\ell\nu$. In fact, even if the hadronic uncertainties related to f_B and V_{ub} are much larger that those for f_K and V_{us}, they cannot hide in any way the huge NP effects in $B\to\ell\nu$ that can arise in our scenario.

5.2.3 $B_s \to \mu^+\mu^-$

The SM prediction for $\mathcal{B}(B_s\to\mu^+\mu^-)$ is $\mathcal{B}(B_s\to\mu^+\mu^-)_{\rm SM} = (3.37\pm0.31)\times10^{-9}$. This value should be compared to the present 95% C.L. upper bound from CDF, BR$(B_s\to\mu^+\mu^-)_{\rm exp} < 5.8\times10^{-8}$ that still leaves a large room for NP contributions. In particular, the MSSM with large $\tan\beta$ allows, in a natural way, large differences between SM and SUSY expectations for $\mathcal{B}(B_s\to\mu^+\mu^-)$ [30]. The SUSY contributions can be summarized by the approximate formula

$$\mathrm{BR}(B_s\to\mu^+\mu^-) \simeq \frac{5\times10^{-8}}{\left[1+0.5\times\frac{\tan\beta}{50}\right]^4}\left[\frac{\tan\beta}{50}\right]^6\left(\frac{500\mathrm{GeV}}{M_A}\right)^4\left(\frac{\epsilon_Y}{3\times10^{-3}}\right)^2 \tag{26}$$

where $\epsilon_Y \simeq -1/16\pi^2 \times A_t/\mu \times H_2(y_{u_R}, y_{u_L})$ with $y_{q_{R,L}} = M^2_{\tilde q_{L,R}}/|\mu|^2$, $H_2(1,1) = -1/2$, $H_2(x\gg1, y=x) \simeq -1/x$ and $H_2(x\ll1, y=x)\simeq 1+\log x$; thus, $\epsilon_Y^{\tilde\chi^-} \sim 3\times10^{-3}$ holds in the limit of all the SUSY masses and A_t equal. As we can see, the present CDF

upper bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$ already provides constraints in some regions of the SUSY parameter space. Moreover, we remind that, although also ΔM_s is a New Physics sensitive observable in the scenario we are considering, it doesn't provide any further constraints in addition to those inferred by the B-physics observables that we have already discussed.

5.3 $(g-2)_\mu$

The possibility that the anomalous magnetic moment of the muon [$a_\mu = (g-2)_\mu/2$], which has been measured very precisely in the last few years [31], provides a first hint of physics beyond the SM has been widely discussed in the recent literature. Despite substantial progress both on the experimental and on the theoretical sides, the situation is not completely clear yet (see Ref. [32] for an updated discussion).

Most recent analyses converge towards a 3σ discrepancy in the 10^{-9} range [32]:

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} \approx (3 \pm 1) \times 10^{-9} \ . \tag{27}$$

Recently, Passera et al. [33] have considered the possibility that the present discrepancy between experiment and the Standard Model (SM) prediction for $(g-2)_\mu$ may arise from errors in the determination of the hadronic leading-order contribution to the latter. If this is the case, the authors of Ref. [33] find a decrease on the electroweak upper bound on the SM Higgs boson mass. By means of a detailed analysis they conclude that this solution of the muon $(g-2)_\mu$ discrepancy is unlikely in view of current experimental error estimates.

The main SUSY contribution to $a_\mu^{\rm MSSM}$ is usually provided by the loop exchange of charginos and sneutrinos. The basic features of the supersymmetric contribution to a_μ are correctly reproduced by the following approximate expression:

$$\frac{a_\mu^{\rm MSSM}}{1 \times 10^{-9}} \approx 1.5 \left(\frac{\tan\beta}{10}\right) \left(\frac{300 \ {\rm GeV}}{m_{\tilde{\nu}}}\right)^2 \left(\frac{\mu M_2}{m_{\tilde{\nu}}^2}\right) \ , \tag{28}$$

which provides a good approximation to the full one-loop result [34].

The most relevant feature of Eqs. (28) is that the sign of $a_\mu^{\rm MSSM}$ is fixed by the sign of the μ term so that the $\mu > 0$ region is strongly favored.

6 LFU vs LFV and the $(g-2)_\mu$ anomaly

As we have previously seen, sizable LFU breaking effects can be generated in SUSY through LFV interactions which involve the third generation. Hence, a legitimate worry is whether the bounds on LFV tau decays, like $\tau \to eX$ (with $X = \gamma, \eta, \mu\mu$), are respected in the region of SUSY parameter space leading to a strong enhancement of the LFU violation [8]. The present and projected bounds (to be achieved at a super B factory) on

	Process	Present Bounds	Expected Future Bounds
(1)	BR($\tau \to e, \gamma$)	9.4×10^{-8}	$\mathcal{O}(10^{-8})$
(2)	BR($\tau \to e, e, e$)	2.0×10^{-7}	$\mathcal{O}(10^{-8})$
(3)	BR($\tau \to e, \mu, \mu$)	2.0×10^{-7}	$\mathcal{O}(10^{-8})$
(4)	BR($\tau \to e, \eta$)	4.5×10^{-8}	$\mathcal{O}(10^{-8})$

Table 1: Present and Upcoming experimental limits on various leptonic processes at 90% C.L.

some of these processes are summarized in Table 1. The most sensitive probe of Higgs mediated effects is generally provided by $\tau \to \ell_j \eta$ [35]; the corresponding branching ratio is given by [36, 37]

$$\frac{Br(\tau \to l_j \eta)}{Br(\tau \to l_j \bar{\nu}_j \nu_\tau)} \simeq 18\pi^2 \left(\frac{f_\eta^8 m_\eta^2}{m_\tau}\right)^2 \left(1 - \frac{m_\eta^2}{m_\tau^2}\right)^2 \left(\frac{|\Delta^{3j}|^2 t_\beta^6}{m_A^4}\right) \tag{29}$$

where $m_\eta^2/m_\tau^2 \simeq 9.5 \times 10^{-2}$ and the relevant decay constant is $f_\eta^8 \sim 110\text{MeV}$. Moreover, $|\Delta^{3j}|^2 = |\Delta_L^{3j}|^2 + |\Delta_R^{3j}|^2$, where Δ_L^{3j} has a similar expression to Δ_R^{3j} [36] and it is such that $\Delta_L^{3j} \sim \delta_{LL}^{3j}$. We note that, in order to generate a non-vanishing $\Delta r_{K\,Susy}^{e-\mu}$, RR-type flavor structures are unavoidable; on the contrary, $Br(\tau \to e\eta)$ can be generated by both LL and/or RR-type mixing angles, being $Br(\tau \to e\eta) \sim |\Delta_L^{31}|^2 + |\Delta_R^{31}|^2$. Given that $\Delta r_{K\,Susy}^{e-\mu}$ and $Br(\tau \to e\eta)$ have the same SUSY dependence, the upper bound on $Br(\tau \to eX)$ is automatically found once we saturate the allowed range (at the % level) for New Physics contributions in $\Delta r_{K\,Susy}^{e-\mu}$. We find that

$$Br(\tau \to e\eta) \simeq 10^{-2} \left(\frac{|\Delta^{31}|^2 t_\beta^6}{m_A^4}\right) \simeq 10^{-8} \times \Delta r_{K\,Susy}^{e-\mu}\,, \tag{30}$$

where the last equality holds when $\Delta_L^{31} = 0$. So, employing the constraints for $\Delta r_{K\,Susy}^{e-\mu}$ at the % level, we obtain $Br(\tau \to e\eta) \leq 10^{-10}$. We conclude that, the present and expected experimental upper bounds on $Br(\tau \to e\eta)$ (see Table 1) still allow large effects in $\Delta r_{K\,Susy}^{e-\mu}$.

On the other hand, $\tau \to \ell_j \gamma$ is the most sensitive probe of LFU violation induced by SUSY gauge mediated effects.

In contrast to the $Br(\tau \to e\eta)$ case, it is not possible to link $Br(\tau \to e\gamma)$ and $\Delta r_{K\,Susy}^{e-\mu}$ in a way that is independent of the specific choice for the susy breaking sector. In particular, as discussed before, the New Physics contributions to $\Delta r_{K\,Susy}^{e-\mu}$ decouple with the heavy Higgs mass m_H, while $Br(\tau \to e\gamma)$ decouples with the heaviest SUSY particle mass $\tilde{m}$ circulating in the gaugino/slepton loop.

In the following, to get a feeling of where we stand, we will evaluate $Br(\tau \to e\gamma)$ in the region of the parameter space where large LFU breaking effects in $\Delta r_{K\,Susy}^{e-\mu}$ can

be generated. In particular, a necessary ingredient in order to get large $\Delta r_{K\,Susy}^{e-\mu}$ values is to maximize the size of the effective LFV coupling Δ_R^{31} (remember that $\Delta r_{K\,Susy}^{e-\mu} \sim t_\beta^6/M_H^4 \times \Delta_R^{31}$) and this happens when $\mu \gg \tilde{m}$. In this latter case, starting from the full expressions of Ref. [38], we find the following expression

$$\frac{BR(\tau \to \ell_j \gamma)}{BR(\tau \to \ell_j \nu_\tau \bar{\nu}_j)} \simeq \frac{\pi \alpha_{el}}{3G_F^2} \left(\frac{\alpha_Y}{4}\right)^2 \left(\left|\delta_{LL}^{3j}\right|^2 + \left|\delta_{RR}^{3j}\right|^2\right) \frac{\mu^2}{\tilde{m}^2} \frac{t_\beta^2}{\tilde{m}^4}\,. \tag{31}$$

From Eq. 31 we can get

$$BR(\tau \to \ell_j \gamma) \approx 5 \times 10^{-8} \left(\left|\frac{\delta_{RR}^{3j}}{0.5}\right|^2 + \left|\frac{\delta_{LL}^{3j}}{0.5}\right|^2\right) \left(\frac{t_\beta}{50}\right)^2 \left(\frac{1\ \text{TeV}}{\tilde{m}}\right)^4 \frac{\mu^2}{\tilde{m}^2}\,, \tag{32}$$

showing that large (order one) mixing angles for $\delta_{LL,RR}^{3j}$ are phenomenologically allowed, provided there exists a rather heavy spectrum for the soft sector. For instance, for $\mu/\tilde{m} = 4$, it turns out that $\tilde{m} \geq 2$TeV. Obviously, such a lower bound on $\tilde{m}$ can be relaxed for smaller values of $\delta_{LL,RR}^{3j}$ and/or $\tan\beta$. Moreover, one can easily find the following approximate expression

$$\Delta r_{K\,Susy}^{e-\mu} \leq 10^{-1} \times \frac{BR(\tau \to e\gamma)}{10^{-7}} \left(\frac{\tilde{m}/M_H}{4}\right)^4 \left(\frac{t_\beta}{50}\right)^4\,, \tag{33}$$

showing that, in the large $\tan\beta$ regime and for heavy Higgs masses lighter than those of the soft breaking terms, experimentally visible LFU breaking effects in $K \to \ell\nu$ can be naturally obtained. In particular, large LFU breaking effects even above the 10% level (already excluded experimentally), can be always compatible with the experimental constraints on $BR(\tau \to e\gamma)$ for slepton/gaugino masses at the TeV scale. However, we stress again that it is not possible to correlate $BR(\tau \to e\gamma)$ to $\Delta r_{K\,Susy}^{e-\mu}$, unless specific SUSY breaking mechanisms (relating $\tilde{m}$ and M_H) are assumed.

On the contrary, the processes $\ell_i \to \ell_j \gamma$ are intimately linked to the muon anomalous magnetic moment $(g-2)_\mu$, as they both arise from dipole transitions [39].

Thus, in the following, we address the interesting question of whether it is possible, within SUSY theories, to account for the current $(g-2)_\mu$ anomaly, while generating, at the same time, LFU breaking effects in $\Delta r_{K\,Susy}^{e-\mu}$ at the % level. As we will show, the answer is positive and this will lead to set a lower bound on $BR(\tau \to e\gamma)$. To see this point explicitly, let us derive the correlation between $BR(\ell_i \to \ell_j \gamma)$ and $(g-2)_\mu$ for the relevant case where $\mu/\tilde{m} \gg 1$; in this case, Δa_μ is well approximated by the expression

$$\begin{aligned}\Delta a_\mu &\simeq \frac{\alpha_Y}{24\pi} \frac{\mu}{\tilde{m}} \frac{m_\mu^2}{\tilde{m}^2} t_\beta \\ &\simeq 3 \times 10^{-9} \left(\frac{\mu/\tilde{m}}{5}\right) \left(\frac{400\text{GeV}}{\tilde{m}}\right)^2 \left(\frac{t_\beta}{50}\right)\end{aligned} \tag{34}$$

and thus we find that

$$
\begin{aligned}
BR(\tau \to \ell_j \gamma) &\simeq \frac{12\pi^3}{m_\mu^4}\left(\frac{\alpha_{el}}{G_f^2}\right)(\Delta a_\mu)^2 \left(\left|\delta_{RR}^{3j}\right|^2 + \left|\delta_{LL}^{3j}\right|^2\right) BR(\tau \to \ell_j \nu_\tau \bar{\nu}_j) \\
&\simeq 3 \times 10^{-9} \left(\frac{\Delta a_\mu}{1 \times 10^{-9}}\right)^2 \left(\left|\frac{\delta_{RR}^{3j}}{0.01}\right|^2 + \left|\frac{\delta_{LL}^{3j}}{0.01}\right|^2\right) . \qquad (35)
\end{aligned}
$$

From Eqs. 33, 34, 35 we conclude that, LFU breaking effects at the % level, typically implying $\delta_{RR}^{31} \geq 0.01$, are compatible with the current experimental bounds on $BR(\tau \to e\gamma)$; moreover, if we additionally require that SUSY effects explain the current discrepancy for the muon anomalous magnetic moment, i.e. $\Delta a_\mu \geq 1 \times 10^{-9}$ at the 2σ level [32], LFU breaking effects at the % level unavoidably imply large effects in $BR(\tau \to e\gamma) \geq 3 \times 10^{-9}$, well within the expected reach of a superB factory.

As we have seen, sizable LFU breaking effects originating from LFV interactions require a flavor mixing in the 13 sector. Thus, from a phenomenological point of view, $\Delta r_{K\,Susy}^{e-\mu}$ is naturally related to $\tau - e$ transitions as, for instance, $\tau \to e\eta$, $\tau \to e\gamma$ etc. However, a legitimate question that can be addressed is what one would expect for $\tau - \mu$ and $\mu - e$ transitions when sizable sources of LFV in the $\tau - e$ sector are assumed.

In particular, from a model building point of view, it seems hard to generate large effects for $\tau - e$ transitions while keeping the effects for $\tau - \mu$ transitions small. Moreover, once $\tau - \mu$ transitions are induced, an effective $(\mu - e)_{eff.}$ transition of the type $(\mu - e)_{eff.} = (\mu - \tau) \times (\tau - e)$ is also induced and processes like $\mu \to e\gamma$ are unavoidable.

In the following, we will address the above issue more quantitatively. In particular, the analogue expression of Eq. 31 for the $\mu \to e\gamma$ case reads

$$
\frac{BR(\mu \to e\gamma)}{BR(\mu \to e\nu_\mu \bar{\nu}_e)} = \frac{\pi \alpha_{el}}{3G_F^2}\frac{t_\beta^2}{\tilde{m}^4}\left(\frac{\alpha_Y}{10}\right)^2 \left|\delta_{RR}^{23}\delta_{RR}^{31} + \frac{m_\tau}{m_\mu}\delta_{LL}^{23}\delta_{RR}^{31}\right|^2 \left(\frac{\mu}{\tilde{m}}\right)^2 , \qquad (36)
$$

where, besides the combination of LFV sources relevant for the present discussion, i.e. RR-type LFV sources, we have also kept the $\delta_{LL}^{23}\delta_{RR}^{31}$ contribution. In fact, this last contribution is enhanced, at the amplitude level, by the ratio m_τ/m_μ compared to the $\delta_{RR}^{23}\delta_{RR}^{31}$ contribution and thus, potentially large even when $\delta_{LL}^{23} < \delta_{RR}^{32}$. Finally, it turns out that

$$
BR(\mu \to e\gamma) \simeq 10^{-11}\left|\frac{\delta_{RR}^{23}\delta_{RR}^{31}}{10^{-2}} + \frac{m_\tau}{m_\mu}\frac{\delta_{LL}^{23}\delta_{RR}^{31}}{10^{-2}}\right|^2 \left(\frac{t_\beta}{50}\right)^2 \left(\frac{1\,\mathrm{TeV}}{\tilde{m}}\right)^4 \left(\frac{\mu}{\tilde{m}}\right)^2 . \qquad (37)
$$

Eq. 37 shows that it is not possible (in the large $\tan\beta$ regime, at least) to have simultaneously order one MIs δ_{RR}^{23} and δ_{RR}^{31}, unless we push $\tilde{m}$ in the multi-TeV regime. In particular, if $\delta_{RR}^{23} = \delta_{RR}^{31} = 0.1$, $\delta_{LL}^{23} = 0$ and $\mu/\tilde{m} = 4$, it turns out that $\tilde{m} \geq 2$TeV. Clearly, such a scenario is not compatible with an explanation of the $(g-2)_\mu$ anomaly.

Moreover, as we have discussed in the previous sections, there is also the possibility to obtain negative values for $\Delta r_{K\,Susy}^{e-\mu}$ (see Eq. 12) when both RR and LL-type of flavor

violating sources for the $1-3$ transition are present. However, this possibility can be constrained, in some cases, by the experimental upper bounds on $BR(\mu \to e\gamma)$. In fact, combining Eq. 12 with Eq. 36, it turns out that

$$|\Delta r^{e-\mu}_{K\,Susy}| \leq 3 \times 10^{-3} \times \sqrt{\frac{BR(\mu \to e\gamma)}{10^{-11}}} \left(\frac{\tilde{m}/M_H}{10}\right)^2 \left(\frac{t_\beta}{50}\right)^2 \left|\frac{\delta^{31}_{LL}}{\delta^{32}_{LL}}\right| \,, \tag{38}$$

thus, unless we assume $\delta^{32}_{LL}/\delta^{31}_{LL} << 1$ (that is typically unnatural from a model building point of view), we are lead with large effects in $BR(\mu \to e\gamma)$ even for an heavy soft sector at the TeV scale. However, we stress that from a pure phenomenological perspective, $BR(\mu \to e\gamma)$ doesn't impose a direct bound on $\Delta r^{e-\mu}_{K\,Susy}$ given that different parameters enter the two quantities.

Let us finally point out that, when $\delta_{RR} >> \delta_{LL}$, the following upper bound on $\Delta r^{e-\mu}_{K\,Susy}$ holds

$$\Delta r^{e-\mu}_{K\,Susy} \leq \frac{Br(\tau \to \mu\eta)}{10^{-8}} \times \frac{BR(\tau \to e\gamma)}{BR(\tau \to \mu\gamma)} \,. \tag{39}$$

Clearly, only the discovery of LFV signals in some of the above observables, by means of improved experimental sensitivities, would shed light on the scenarios outlined before.

We conclude this section pointing out that the same New Physics effect observable in the Kaon system through $\Delta r^{e-\mu}_{K\,Susy}$ is also observable, in principle, in B physics systems by means of purely leptonic decays of charged B meson. In particular, it is found that

$$\frac{BR(B \to e\nu)}{BR(B \to e\nu)_{SM}} = \left[1 + \frac{m^4_B}{m^4_K}\,\Delta r^{e-\mu}_K\right] \simeq \left[1 + 10^2 \times \left(\frac{\Delta r^{e-\mu}_K}{10^{-2}}\right)\right] . \tag{40}$$

This means that, a LFU breaking effect at the % level in the $K\ell 2$ systems, implies an enhancement of two orders of magnitude in $BR(B \to e\nu)$ compared to its SM expectation. However, given that $BR(B \to e\nu)_{SM} \approx 10^{-11}$, an experimental sensitivity at the level of $BR(B \to e\nu)_{exp.} \leq 10^{-9}$ would be necessary.

7 Numerical Analysis

In the following, we will analyze the allowed size for the LFU breaking effects in $R^{e/\mu}_P$ both in the lepton flavor conserving and violating cases.

In the former case, LFU breaking effects arise from mass splittings between sleptons of the first and second families ($m_{L1(L2)}$, $m_{R1(R2)}$), as discussed in Sec. 4. In Fig. 2, we perform a numerical analysis of the allowed values for $\Delta r^{e/\mu}_K$ (see Eq.15) through a scan over the following SUSY parameter space: ($m_{L1(L2)}$, $m_{R1(R2)}$, $m_{\tilde{Q}}$, $m_{\tilde{g}}$, $m_{\tilde{W}}$, $m_{\tilde{B}}$, M_H) $<$ 2.5TeV, $\mu <$ 5TeV and $\tan\beta < 60$. In particular, we allow different entries for the left-left (LL) and the right-right (RR) blocks in the slepton mass matrix for the first two generations, i.e. for $m_{L1(L2)}$ and $m_{R1(R2)}$ respectively. Moreover, we also

impose all the constraints discussed in Sec. 5. In Fig 2, on the left, we show $\Delta r_K^{e/\mu}$ as a function of the (left-handed) mass splitting between the second and first slepton generations. Black dots refer to the points satisfying the $(g-2)_\mu$ discrepancy at the 95% C.L., i.e. $1 \times 10^{-9} < \Delta a_\mu < 5 \times 10^{-9}$.

As we can see, the maximum LFU breaking effects are reached for maximum mass splitting between sleptons. However, when $m_{L1} = m_{L2}$, we would expect LFU breaking effects going to zero, in contrast to what is shown by Fig 2. This happens because mass splittings for right-handed sleptons $m_{R1} \neq m_{R2}$ (not explicitly visible in Fig 2), can still generate LFU breaking effects even in the case where $m_{L1} = m_{L2}$. We see that values for $|\Delta r_K^{e/\mu}|$ as large as 5×10^{-3} are possible for slepton masses splitted by a factor 10. However, potentially visible values for $|\Delta r_K^{e/\mu}|$ of order of $\sim 2 \times 10^{-3}$ are obtained even for smaller mass splittings, i.e. for $m_{L1,L2}/m_{L2,L1} \sim 2$.

Interestingly enough, the sign of these LFU breaking effects depends on the ratio between the slepton masses. In particular, if the left-handed smuons are heavier then the selectrons $\Delta r_K^{e/\mu} > 0$, while $\Delta r_K^{e/\mu} < 0$ if the smuons are lighter then the selectrons. The opposite situation happens for mass splittings of right-handed smuons.

In Fig 2, on the right, we show the regions of the parameter space in the $\tan\beta - M_H$ plane where $0.001 < |\Delta r_K^{e/\mu}| < 0.003$ (red dots), $0.003 < |\Delta r_K^{e/\mu}| < 0.005$ (black dots) and $|\Delta r_K^{e/\mu}| > 0.005$ (yellow dots). We observe that the narrow region where $m_H \leq 200$GeV corresponds to the points where the $B \to \tau\nu$ constraints are not effective. This does not occur not because the new physics contributions to $B \to \tau\nu$ are small; quite the contrary, the reason is that they are quite large ($\sim 2\times$ SM ones) and destructively interfere with the SM contribution (see Eq. 25). On the other hand, the region of the $\tan\beta - M_H$ plane between the two allowed areas is excluded by the $B \to \tau\nu$ constraint.

Let us now discuss LFU breaking effects in $\Delta r_K^{e/\mu}$ as generated by LFV contributions stemming, in particular, from RR-type flavor violating sources only. In Fig. 3, on the left, we report $\Delta r_K^{e/\mu}$ as a function of $\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to e\eta)$ while, on the right, we report $\Delta r_K^{e/\mu}$ as a function of M_H. The plots have been obtained by means of a scan over the following parameter space: $(m_{L,R}, m_{\tilde{Q}}, m_{\tilde{g}}, m_{\tilde{W}}, m_{\tilde{B}}, M_H) < 2.5$TeV, $\mu < 5$TeV, $|\delta_{RR}| < 0.5$, $|\delta_{LL}| = 0$ and $\tan\beta < 60$ and imposing all the constraints discussed in Sec.5. Black dots refer to the points satisfying the $(g-2)_\mu$ anomaly at the 95% C.L., i.e. $1 \times 10^{-9} < \Delta a_\mu < 5 \times 10^{-9}$. Fig. 3 clearly shows that there are quite a lot of points in the interesting region where $0.001 < \Delta r_K^{e/\mu} < 0.01$ accounting for the $(g-2)_\mu$ anomaly and that are compatible with the experimental constraints of $\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to e\eta)$.

As discussed in the previous section and as it is shown in Fig. 3, $\Delta r_K^{e/\mu}$ and $\mathcal{B}(\tau \to e\eta)$ are closely related, at least in the limiting case where $\Delta_L = 0$. On the contrary, an analogue correlation between $\Delta r_K^{e/\mu}$ and $\mathcal{B}(\tau \to e\gamma)$ is absent, due to their different dependence on the SUSY mass spectrum.

We also emphasize that experimentally visible effects in $\Delta r_K^{e/\mu}$ (at the 0.1% level) can be reached up to charged Higgs masses at the TeV scale, as shown in Fig. 3 on the

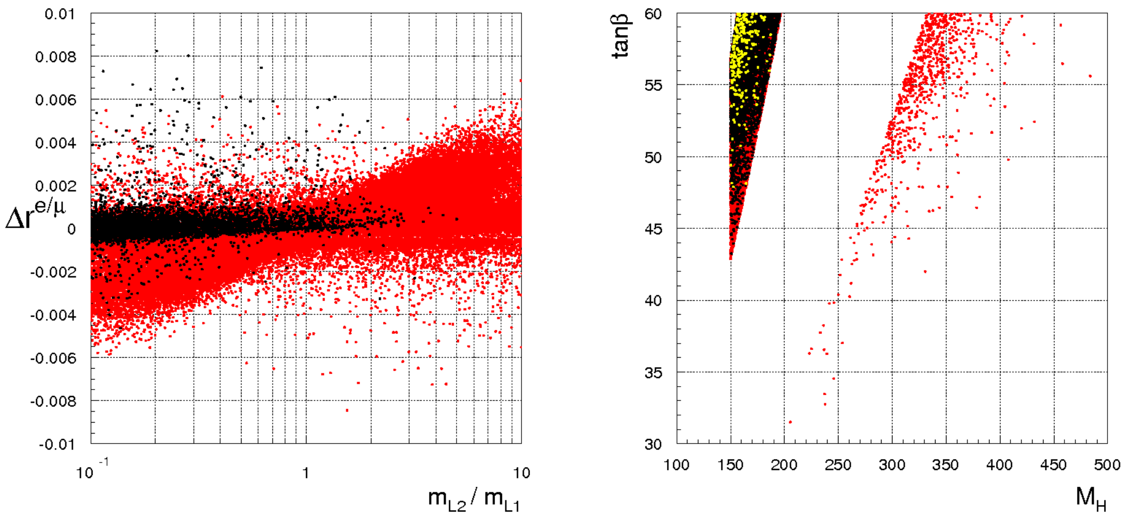

Figure 2: SUSY Lepton Flavor Conserving contributions to $\Delta r_K^{e/\mu}$. Left: $\Delta r_K^{e/\mu}$ as a function of the (left-handed) mass ratio between the second and the first slepton generations. Black dots refer to the points satisfying $1 \times 10^{-9} < \Delta a_\mu < 5 \times 10^{-9}$. Right: regions of the parameter space in the $\tan\beta - M_H$ plane where $0.001 < |\Delta r_K^{e/\mu}| < 0.003$ (red dots), $0.003 < |\Delta r_K^{e/\mu}| < 0.005$ (black dots) and $|\Delta r_K^{e/\mu}| > 0.005$ (yellow dots). The plot has been obtained by means of a scan over the following parameter space: $(m_{L_{1,2}}, m_{R_{1,2}}, m_{\tilde{Q}}, m_{\tilde{g}}, m_{\tilde{W}}, m_{\tilde{B}}, M_H) < 2.5$TeV, $\mu < 5$TeV and $\tan\beta < 60$. All the dots present in these and subsequent figures satisfy all the constraints discussed in Sec. 5.

right. Moreover, we also stress that the present experimental bounds on $\Delta r_K^{e/\mu}$ at the % level already set constraints on the SUSY parameter space. In Fig. 4, we show the SUSY parameter space in the $\tan\beta - M_H$ plane probed by an experimental resolution on $\Delta r_K^{e/\mu}$ up to the 0.1% level. In particular, red dots refer to the points satisfying $0.001 < |\Delta r_K^{e/\mu}| < 0.003$, black dots refer to the points where $0.003 < |\Delta r_K^{e/\mu}| < 0.005$ and, finally, yellow dots are relative to the points where $|\Delta r_K^{e/\mu}| > 0.005$. As discussed in Sec. 3, it is also possible to generate LFU breaking effects in $\Delta r_K^{e/\mu}$ by means of a double source of LFV that, as a final result, preserve the lepton flavor (see Eq. 12). This is the case when both LL and RR flavor violating sources are simultaneously non vanishing. The major novelty arising from this last possibility is that now the new physics contributions can interfere with the SM ones; thus, we can get both positive and negative values for $\Delta r_K^{e/\mu}$. This is clearly shown by Fig. 5 that is the analog of Fig. 3 but in the presence of non vanishing δ_{LL} LFV terms. We see that $\Delta r_K^{e/\mu}$ can lie in the experimentally interesting region while satisfying all the current constraints. We observe that also in this case, the requirement of large LFU breaking effects in $\Delta r_K^{e/\mu}$ at the level of $0.001 < |\Delta r_K^{e/\mu}| < 0.01$, can be compatible with an explanation for the $(g-2)_\mu$ anomaly while satisfying the constraints from $\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to e\eta)$.

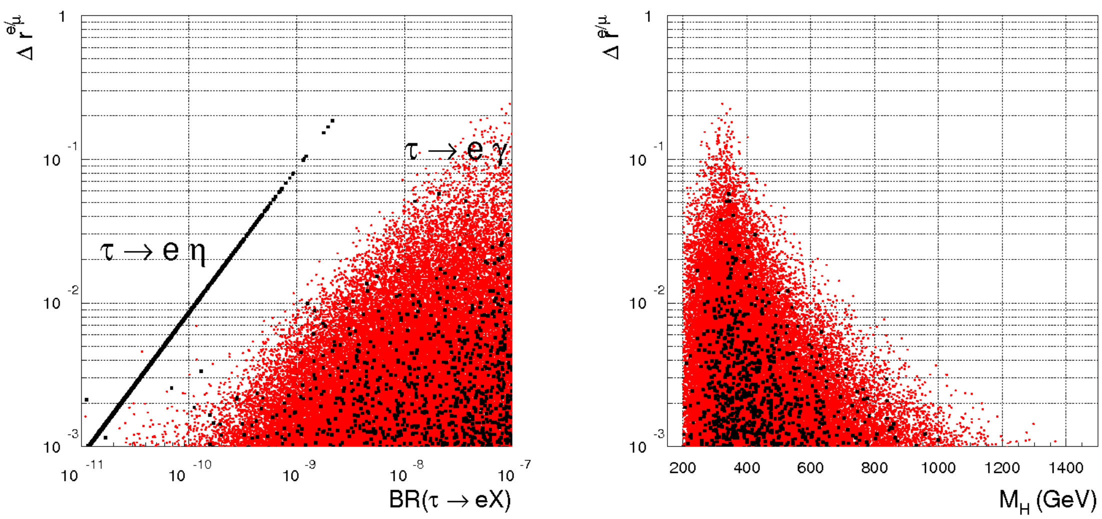

Figure 3: Left: $\Delta r_K^{e/\mu}$ as a function of $\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to e\eta)$. Right: $\Delta r_K^{e/\mu}$ as a function of M_H. Both plots have been obtained by means of a scan over the following parameter space: $(m_{L,R}, m_{\tilde{Q}}, m_{\tilde{g}}, m_{\tilde{W}}, m_{\tilde{B}}, M_H) < 2.5\text{TeV}$, $\mu < 5\text{TeV}$, $|\delta_{RR}| < 0.5$, $|\delta_{LL}| = 0$ and $\tan\beta < 60$. Black dots refer to the points satisfying $1 \times 10^{-9} < (g-2)_\mu < 5 \times 10^{-9}$.

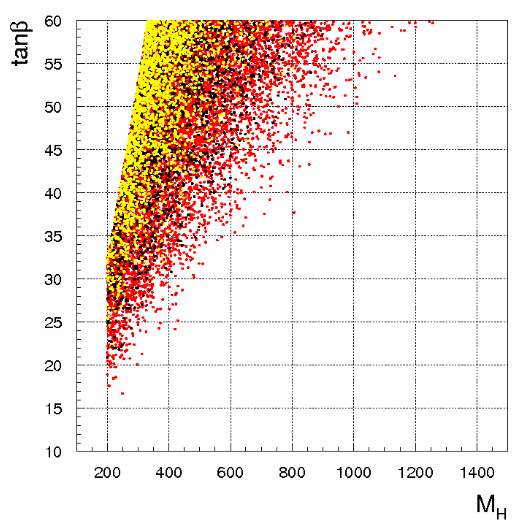

Figure 4: Regions of the parameter space in the $\tan\beta - M_H$ plane where $0.001 < |\Delta r_K^{e/\mu}| < 0.003$ (red dots), $0.003 < |\Delta r_K^{e/\mu}| < 0.005$ (black dots) and $|\Delta r_K^{e/\mu}| > 0.005$ (yellow dots) as obtained by means of the same scan performed in Fig. 3.

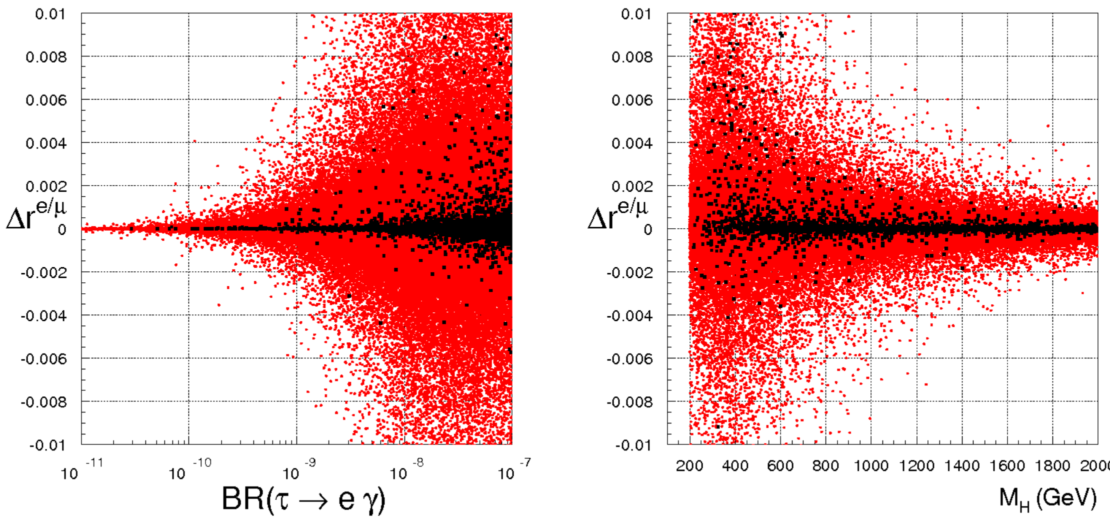

Figure 5: Left: $\Delta r_K^{e/\mu}$ as a function of $\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to e\eta)$. Right: $\Delta r_K^{e/\mu}$ as a function of M_H. Both plots have been obtained by means of a scan over the following parameter space: $(m_{L,R}, m_{\tilde{Q}}, m_{\tilde{g}}, m_{\tilde{W}}, m_{\tilde{B}}, M_H) < 2.5$TeV, $\mu < 5$TeV, $|\delta_{RR,LL}| < 0.5$ and $\tan\beta < 60$. Black dots refer to the points satisfying $1 \times 10^{-9} < (g-2)_\mu < 5 \times 10^{-9}$.

Finally, Fig. 6 shows the parameter space in the $\tan\beta - M_H$ plane probed by an experimental resolution on $\Delta r_K^{e/\mu}$ up to the 0.1% level in analogy to Fig. 4.

8 2HDM framework

Theories with only one Higgs doublet, like the Standard Model (SM), do not contain flavor violating interactions of the fermions with the Higgs bosons. In particular, it is always possible to simultaneously diagonalize the fermion mass matrices and the Higgs-fermion couplings. In general, this is no longer true in models with several Higgs doublets. In fact, up and down-type fermions can couple, at the same time, to more than a single scalar doublet and this naturally leads to FCNC effects at the tree level. To suppress tree level FCNC in the theory so as not to be in conflict with known experimental limits, an *ad hoc* discrete symmetry is typically invoked. For instance, in the 2HDM, the up-type and the down-type quarks couple either to the same Higgs doublet (this is known as the 2HDM-I) [6], or to different doublets (2HDM-II) [6]. On the other hand, in the most general case, the so-called 2HDM-III [40], no discrete symmetries are assumed and FCNC phenomena naturally appear.

The Lagrangian for the LFV Yukawa couplings of the 2HDM type III reads [40]

$$-\mathcal{L} = \eta_{ij}\bar{\ell}_{iL}H_1\ell_{jR} + \xi_{ij}\bar{\ell}_{iL}H_2\ell_{jR} + \text{h.c.} \tag{41}$$

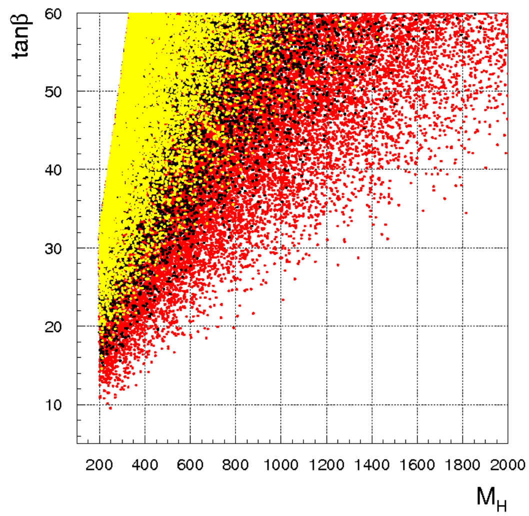

Figure 6: Regions of the parameter space in the $\tan\beta - M_H$ plane where $0.001 < |\Delta r_K^{e/\mu}| < 0.003$ (red dots), $0.003 < |\Delta r_K^{e/\mu}| < 0.005$ (black dots) and $|\Delta r_K^{e/\mu}| > 0.005$ (yellow dots) as obtained by means of the same scan performed in Fig. 5.

where $H_{1,2}$ are the Higgs doublets defined by $H_1 = (\phi_1^+ \ \phi_1^0)$ and $H_2 = (\phi_2^+ \ \phi_2^0)$ while η_{ij} and ξ_{ij} are off-diagonal 3×3 matrices in the flavor space and i, j are family indices.

Passing to the basis where the leptons are in mass eigenstates and expressing the leptonic Lagrangian in terms of ξ_{ij}, one find that

$$\mathcal{L} = -\frac{m_i}{vc_\beta}\bar{\ell}_i\ell_i(s_\alpha h^0 - c_\alpha H^0) + i\frac{m_i t_\beta}{v}\bar{\ell}_i\gamma_5\ell_i A^0 + \frac{m_i t_\beta}{\sqrt{2}v}\bar{\nu}_i(1+\gamma_5)\ell_i H^+ \tag{42}$$

$$- \frac{m_i}{vc_\beta}\bar{\ell}_i\xi_{ij}\ell_j\left[c_{\alpha-\beta}h^0 + s_{\alpha-\beta}H^0\right] - \frac{im_i}{vc_\beta}\bar{\ell}_i\xi_{ij}\gamma_5\ell_j A^0 - \frac{m_i}{\sqrt{2}vc_\beta}\bar{\nu}_i\xi_{ij}(1+\gamma_5)\ell_j H^+ + \text{h.c.} \tag{43}$$

where $v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-1/2} = 246$ GeV. Note that the Lagrangian (43), expressed in terms of the ξ_{ij} matrices, has the same lepton flavor conserving interactions of the to 2HDM-II. To be consistent with experimental data on FCNC processes, Cheng and Sher (CS), inspired by the hierarchy in the fermion masses, have proposed the following famous ansatz for the couplings ξ_{ij} [41]:

$$\xi_{ij} = \lambda_{ij}\sqrt{\frac{m_j}{m_i}}, \tag{44}$$

where the residual arbitrariness of flavor changing couplings is expressed by the parameters λ_{ij} which is constrained by experimental bounds on LFV processes.

By making use of the effective Lagrangion of Eq. 43, it is straightforward to compute

the expression for the quantity $\Delta r_K^{e/\mu}$. It turns out that

$$\Delta r_K^{e/\mu} \simeq \left(\frac{m_K^4}{M_H^4}\right)\left(\frac{m_\tau}{m_e}\right)^2 |\xi_{31}|^2 t_\beta^4 \tag{45}$$

$$\simeq 10^{-2} \times \left(\frac{500\text{GeV}}{M_H}\right)^4 \left(\frac{t_\beta}{40}\right)^4 |\lambda_{31}|^2 , \tag{46}$$

where in Eq. 46 we made use of the the CS ansatz; Eq. 46 clearly shows that a 2HDM of type III, with the addition of the CS ansatz, can naturally predict a LFU breaking in the $K \to \ell\nu$ systems at a visible level for natural values of M_H and t_β. We also remind that the λ_{ij} parameters should be typically of order one [41]. However, once we assume the CS ansatz, we are naturally lead with stringent correlations among all the LFV transitions, as for example $\mu \to e\gamma$, $\mu + N \to e + N$, $\tau \to e\gamma$, $\tau \to \mu\gamma$ and so on.

We have explicitly checked that the precision test provided by LFU breaking effects in $\Delta r_K^{e/\mu}$ represents the most powerful probe of the CS ansatz, at least in the decoupling regime (where $M_H \simeq M_A$ and where the lightest higgs boson doesn't have LFV couplings with fermions) and assuming non vanishing LFV interactions only for $3j$ transitions.

Irrespective of the specific model one can assume, we wish to emphasize that LFU breaking effects in $\Delta r_K^{e/\mu}$ represent the best probe for 31 LFV transitions in a generic 2HDM with tree level LFV couplings (see Eq. 43). To see this point explicitly, it is natural to compare the New Physics sensitivity in LFU violation to that achievable in $\tau \to e\eta$. Indeed, this latter decay channel represents the most sensitive channel to New Physics among all the rare τ decays (this is strictly true in the decoupling limit). It turns out that

$$Br(\tau \to e\eta) \simeq 10^{-8} \times \Delta r_K^{e-\mu} \tag{47}$$

showing that, within a 2HDM-III framework, $\Delta r_K^{e-\mu}$ sets tight constraints on the observation of $\tau \to e\eta$ at the level of $Br(\tau \to e\eta) \leq 10^{-10}$.

9 Conclusions

High precision electroweak tests, such as deviations from the Standard Model expectations of the Lepton Universality breaking, represent a powerful tool to test the Standard Model and, hence, to constrain or obtain indirect hints of new physics beyond it. Kaon and pion physics are obvious grounds where to perform such tests, for instance in the $K \to \ell\nu_\ell$ decays, where $l = e$ or μ.

In this paper, we have analyzed the domain of $\Delta r_K^{e/\mu}$ between $10^{-3} < \Delta r_K^{e/\mu} < 10^{-2}$. An evidence of LFU violation at the level of $\Delta r_K^{e/\mu}$ larger than 5×10^{-3} unambiguously points towards the presence of LFV sources. On the other hand, if our increased experimental sensitivity allows us to observe an LFU violation with values of $\Delta r_K^{e/\mu}$ smaller than 5×10^{-3}, then both the flavor conserving and the flavor changing sources of LFU violation can be at play. In any case, the observation of a non-vanishing $\Delta r_K^{e/\mu}$ in the

range $10^{-3} < \Delta r_K^{e/\mu} < 5 \times 10^{-3}$ would severely limit values in the $M_H - \tan\beta$ plane. If a signal exists at a such a level, the LHC results become the crucial tool to disentangle flavor conserving and flavor changing sources of LFU violation.

Interestingly enough, a process that in itself does not need lepton flavor violation to occur, i.e. the violation of $\mu - e$ non-universality in $K_{\ell 2}$, proves to be quite effective in constraining not only relevant regions of SUSY models where lepton flavor is conserved, but even those where specific lepton flavor violating contributions arise. Indeed, a comparison with analogous bounds coming from τ Lepton Flavor Violation decays shows the relevance of the measurement of R_K to probe Lepton Flavor Violation in SUSY.

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